

Problems

1. to what extent does grad  $f$ , <sup>flow</sup> near a critical pt. depend on the metric
2. topological Rokhlin's thm.
3. "geometric" proof of wk. mixing  $\Rightarrow$  mixing for full set of  $t$
4. class. of smg. by local properties of grad. flow
5. Homogeneous spaces

- a. implications between
1. unique ergodicity
  2. minimality
  3.  $h_u = 0$  and ergodic

- b. <sup>simple or</sup> semi-simple case
1. which one-param. subgps are  $G^u$ 's for ergodic affine
  2. try 2.

relate dyn. properties to repn.

try  
in  
first

- d.  $K \Rightarrow$  Bernoulli
- e. wk. mixing + center s.s.  $\Rightarrow$  Bern.
- f. ergodic  $\Rightarrow$  unique meas. max.  $h(u)$

6. zeta fun. for Ax. A
- top. identification + } Try. (dual  
conj. inv. of  $\mathbb{Z}(0)$  }  $\mathbb{Z}$  first
  - $\zeta(s)$  meromorphic for  $C^\infty$  flow
  - connection with Laplacian vs. geod. results; automorphic forms
  - Anosov actions

7. Structure of basic sets

- classification via  $(R, A)$
- local Ax. A  $\Rightarrow$  embeddable
- can. coord.  $\Rightarrow$  "  
(opt. abel. groups  $\Omega$ 's)
- phantom homology gps.  
- shift equiv. of induced maps!
- $\dim \Omega$ ; within a mfd.

8. Non Ax. A examples - Newhouse, Abraham-Smale, Simon, Kerckhoff, billiards

- axiomatic description
- stat. properties
- $\forall \epsilon > 0 \exists$  horseshoe inside with  $h(f|X_\epsilon) \geq h(f) - \epsilon$
- stat. properties of Leony in particular

e. any specification type property

9. unique ergodicity of  $W^n$  for partially Anosov systems/diffeos
10. Stat. + dynamics of transformations of  $[0, 1]$  - "nontrivial" - exp. like examples
11. Nonalgebraic Anosov diffeos, class. 3-dim Anosov flows, var. neg. cur. surface conj. to const. cur.?
12.  $h(f) \geq \log |A|$ 
  - a. diffeos.
  - b.  $\mathbb{Z}$  finite + hyp.
  - c.  $Ax. A$  with cycles
13. In Parry's "conj. to linear" paper, what are properties of constructed measure? Does this work for equil. states too?
14. Suspensions of diffeos. - generally not a const. time sys? - strongest statement is what. For  $Ax. A$  attractors?

15. Renewal thm. for dependent  
r.v.'s

- a. derive via motivation of  
 $Ax$ ,  $A$  flows mixingness
- b. how fast is the mixing  
for  $Ax$ ,  $A$  flows

16. Brownian motion or diffusion  
given a flow

17. Symbolic dynamics for billiards

18. Interpret  $\log \lambda(x)$  as a  
potential fun? - Kohn. idea  
on surfaces neg. curv.

19. Can you construct some Banach  
space so that  $h_u$  is an eigenvalue  
of some can. operator

20. ds. systems in stat mech.  
- top dyn. formulation?

21.  $\phi_t$  C-dense. ~~Unique  $\mu$  which~~  
~~is  $\phi_t$  inv. max entropy?~~  
 $\forall \phi_t$  - inv  $\Rightarrow \phi_t$  - inv  $\neq$  fact.  $\mu$  on closed  $\text{orb}$

22. canonical embedding of  
 $Ax, A \in \mathbb{R}^2$

23. canonical  $C^0$  perturb. of  
Anosov differ to  $C^0$ -sim  $\mathbb{R}^2$ 's  
with same entropy

24. bifurcation of  $Ax, A$  in  
terms of symbols

(25) coding s.s. finite type  
with same entropy

26. entropy in Hamiltonian  
case. For P.D.E.'s? Relation  
to ODE's?

27. Construct 2 dim. Hamilt.  
differ. with ergodic set  
of  $h_{\text{pos. meas.}}$

28. Kupka-Smale +  $h(f) > 0$   
 $\Rightarrow$  homocline pt.

29. Find  $Ax, A$  inf. attractor  
in some ODE on  $\mathbb{R}^3$  (quadratic)

30. Fixing  $\text{cpt}$  of  $M$ . What is possible  
behaviors of good. flow for  
all Riem. metric.  
e.g. ~~for some of~~  $\pi_1(M) = 0$   
~~has~~ some good. flow have  
entropy 0

31. Anosov diffeo.  
a. hyp. on  $\mathbb{T}^n(M)$   
b. fixed pts.  
c.  $\Omega = M$

~~32. Is even~~

32. Classify s.s. with specification

33. Refine  $P(g)$ , equil. state  
for certain words  $g$ .

34.  $\mu$  equil. state some cls  $g$   
on  $\sum_N \Rightarrow h_\mu > 0$ ?

35. ergodic non hyp. autom  
of  $\mathbb{T}^n$  not part of s.s.?  
finite type? not specification

36. Geod. flow expansive  
 $\Rightarrow$  Anosov flow

37. Geod. flow h-expansive?

38. ~~Let~~  $M = \text{Riem. metric } g$   
with  $g \text{ vol. } M = 1$ .  
What is  $\{h(\text{geod. line } \gamma) : \gamma \in M\}$ ?  
Relation to top. inv. of  $M$

39. Ambrose-Kakutani thm. for  
 $R^n$  ~~flows~~ actions

40. Entropy of autom of  $C^*$  alg.

41. Is  $\Phi_1$  a ctly.  $\Phi$  for  
 $h: \text{Diff} \rightarrow R$  for  $\Phi_t$  Anosov flow?

42. Is  $h(\Phi_t | E)$  diff. in  $E$ ?  
for Hamiltonian case.  
Any Relation to class. or quant. <sup>stat</sup> mech.?

43. Defn. Gibbs near. for  
homeo. - relate to eq.  
state?

44. How big is  $\{u \in \mathbb{R}^d\}$  ~~regul.~~  
state some  $\{e \in C(\Lambda)\}$  is  $M_f(X)$

45. Any "local" invariants  
(near fixed pt.) which  
are entropy like

46. Eg. states for 1-dim  
quan. lattice systems w/o  
finite range

47. Any entropy-like  
inv. for singularity  
of diff. map?

48. Suppose  $F: \text{Cantor set } C \rightarrow \mathbb{R}$   
bdd. total var. Is there a  
homeo.  $g: [0,1] \rightarrow C$  and



(Lip.)  
diff  $f: [0,1] \rightarrow \mathbb{R}$  s.t.  
 $F = f \circ g \circ C$ .

49. Does min. or u.e. for diffeo  
 $\Rightarrow h(f) = 0$  (try hom, cat too)

a. Is there a minimal diffeo  
hom. to  $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \times \text{id. on } T^3$

b (Seifert) min. flow on  $S^3$

50. ~~Is~~ <sup>Is</sup> there a transitive <sup>of Katok</sup>  
ergodic diffeo of  $S^2, D^2$

51. Look for inv. meas. of  
some standard foliations

52. Define  $\Omega(\text{foliation})$ . Does  
 $h(\text{fol.}) > 0$  make sense?

53. Is the space of an expansive  
homeo always finite dim?  
Is every minimal expansive  
homeo. on a 0-dim. space?

54.  $C$ -dense An. A flows
- speed of mixing
  - asympt. expressions for # per. orbits
  - is  $\phi_A$  intrinsically ergodic
  - direct pf. of mixing of meas.
  - analogue of " $h(f) \geq \log |X|$ "
  - understand  $\det(t-A)$  in var. -rel. to  $S(t)$
  - stability of  $C$ -density for attractors
  - cond. on  $g$  so that  $\Sigma_A(g)$   $C^\infty$  or anal. embeddable as basic set
  - can a closed orbit of Anosov flow be null-homotopic

55. Entropy of autom. in algebra  
-  $g_{\mathbb{Z}^2}$ ,  $g_{\mathbb{Z}^3}$

56. horocycle flow  $\Leftrightarrow$  expansive flow

57.  $l(f^n)$  grows slowly with  $n$  for many  $\delta$  and An. A diffeot

58. Is Gutzwiller's example  
an Anosov flow?

59. Computer program for Anosov  
attractors

60. Study flows  $V(r) + \frac{1}{2}mv^2 = H$   
for various  $V(r)$ . Stat.  
mech. literature Henon-Hedley  
Toda...

61. Rokhlin thm. for ctbl.  
pseudo-gp. action  
- ergodic thm. & avg.  
procedures

62. covering space for  $\Sigma_A \rightarrow T^2$   
cover. to  $R^2 \rightarrow T^2$

63.  $C^r$  diffeos. which are not  $C^r$  qualitatively.  
 Find.  $C^1$  diffeo  $f: V \rightarrow V$  and  $C^2$  embedding  $g: V \rightarrow M$  and  $\tilde{f}$  extending  $f$  to  $M$ , s.t.  $\tilde{f}$   $C^2$  on  $M$ ,  $\tilde{f}|_V$  qual not  $C^2$ . (qual behavior of  $\tilde{f}$  due to  $C^1$   $V$  but not exp.  $\tilde{f}$ )

64. Entropy of gp. action.  
 There is no smooth  $R^n/Z^n$  action with pos. entropy when  $n > 1$ . Is this true for all lie gp. (or  $\mathbb{R}$ ) of  $\dim > 1$ ? Nilpotent.

65. Foliation ergodic theory

- Ambrose-Kakutani - esp.  $R^n$
- Does mixing make any sense - use category, diff.  $C^\infty$ , analytic str.
- average procedure difficulties - ergodic thm, existence inv. meas., ergodic decomp, u.e. + unif. conv. - poly. growth unif. thm. an inv.
- had at some specific foliations
- Plante stuff on connections with homology
- positive entropy make sense

66. Central limit thm. <sup>other</sup> ~~state~~  
~~that~~ strong statistics near an  
attractor of diff'er.

67. Correspondence principle  
of quantum mechanics.

Investigate for some  
simple mechanical systems.

Is it experimentally related to <sup>uncertainty</sup> ~~princ.~~ ~~princ.~~ ~~princ.~~?

68. Electric circuits.  
a. Analogue computer for  
finding  $Ax$ . At examples  
b. Is noise sometimes  
due to hyp. set in the  
dynamics?

69. Is  $h: \text{Diff}^r(M) \rightarrow \mathbb{R}$   
generically cts. for  
some  $r$ ?

70. Classify all Anosov  
systems or attractors  
(which  $\Omega^2$  can occur  
as attractors).

71. Cancellation of  $\Omega_i$ 's.  
Simplest  $f$  in an isotopy  
class

~~72. Does  $h(f) = 0$  for a  
cts. (or diff.) flow on  
opt. 2-manifold.~~

73. If  $f$  Anosov and  $g \sim f$ ,  
does  $h(g) \geq h(f)$ ?

74. If  $f \in Ax, A$  is there  
an  $Ax, A$   $g$   $C^0$  near  $f$  with  
 $\dim \Omega(g) = d + h(g) = h(f)$ .

75. Conditions on  $M$  to  
admit Anosov  $f$ .

76. Does  $h(f)$  have minimum  
in isotopy class?

77. Conjugacy between  
top. + measure th.

a. what ~~is~~ s.d.  $h(f)$   
an invariant

b. entropy-conj. +  
equiv on Baire sets  
— what are the equiv-  
relations on homeo  
or maps of  $S^1$ , subshifts

78. Calculate  $h$  for CDF  
on  $\mathbb{R}^n$ , e.g. linear diffeom.

79. infinite measure space  
autom.

80. Really examples of expansive  
diffeom. related to Anosov  
diffeo. Are expansive  
diffeo. likely to be Anosov?

81. geometric th. of u.e. for  
in lat. rot.  $S^1$ .

82. (Plante) codim 1 minimal  
fol. has at most one  
inv. measure.

83. Unstable foliations  
of Anosov diffeo given by  
nilpotent group's action

84. Invariant or approx. in-  
finite dim. subspaces for  
Navier-Stokes eqn.

85. Codon frequencies via  
equiv. state semi-potential"

86. alg. varieties - Weil conj.  
cohomology - any entropy  
here? any rel. to hom.  
equivalences?

87. If translation by gp. elt.  
on  $G/P$  is minimal, is that elt.  
nilpotent in  $g$ ? (i.e. have 0 entropy)



88. Index of fixed pt. of diffeo  
removable by small  
perturbation (Morse).

89. For An-flow  $f_t$  on  $M$ . Try  
to get approx. to curves  $\subset M$   
by piecewise orbits  $f_{t_i}$ , calculate  
 $\int_H M \dots$  as in Morse theory

90. If  $f$  An. on  $M$  and  
 $M$  contractible, what does  
 $H^k(M) \cong H^k(\pi_1(M))$  tell  
you via  $f_*$  eigenvalue info?  
(see Hu ~~p. 200~~ p. 200-202)

91. Among  $C^1$  "expanding  
maps" on  $[0,1]$  is  $h$   
cts? (a.e.)

92. Among deg  $n$  poly. maps  
of  $[0,1] \rightarrow [0,1]$ , and Ax. A ones  
open+dense? bad ones  
stratified set...

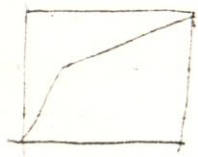
(write Keane)

~~but~~ bouncing disks  
91. 3 particles on  $S^1$  - how do  
you do it.

2 particles on  $T^2$  different  
mass

92.  $a_i$  an integer  $\geq 0$ ,  $a_1 \leq a_2 \leq \dots$   
 $a_i \geq a_{i+k}$  when  $i, j \in \mathbb{N}$   
Can these be the seq. of symbol  
seq lengths 1 to  $n$ ?

93 (Heron)



$f: [0,1] \rightarrow [0,1]$

$$f(x) = f(x) + b$$

Is  $f_b$  cyclic w.r.t. left  
meas. (not inv), w.e.

$$f_b(A) = A \Rightarrow m(A) = 0 \text{ or } 1$$

94. Does every manifold  $M^n$  ( $n \geq 3$ ) admit smooth Borelli flow?

95. (Ruelle) Markov part. for alg. geom. examples

96. (Ruelle) AM flow using me. flows on  $M$  without assuming  $M = T^n$ .

97.  $C^1$  noncyclic ~~system~~ diffeos of  $T^2$  preserving orb. meas.

98. Uie. on hiepp, one with finite area instead of compact hom. spaces

99. (Dang) find ~~for~~ open partition in  $\mathbb{R}^2$  w. Bern. Find inv. of finitistic codes

Symbolic dynamics on  $\mathbb{R}^2$  Markov, Bernoulli, Lowsky

101. If a  $C^1$  Anosov preserves smooth inv.  $\mu$ , is  $\mu$  an equal. state for  $\log |Df|$ ?

102. Is the false zero function of a basic set rational? Define false zero function of a flow basic set?

$$\tilde{z}(s) = \prod_{\lambda \in \Gamma} (1 - \lambda e^{-s \ell(\lambda)})$$

$\pm 1$  according to whether  $Df|_{E_x^u}$  plus or minus orientation

$\tilde{z}(s)$  related to  $z(s)$  related to mod 2 part of torsion

103. How can you write  
 $1+t^2+t^2 = \prod_{i=0}^{\infty} (1+t^{4i})$   
 in  $\mathbb{Z}[[t]]$ .

104. Note:

$$\frac{1}{1-t} = \prod_{i=0}^{\infty} (1+t^{2^i}) \text{ in } \mathbb{Z}[[t]]$$

Is there a map  $D^2 \rightarrow D^2$  s.t.  
 no sinks or sources + finite  
 happy people by this formula?  
 cellularization of  $\frac{1}{1-t}$ ?

105. Sullivan's stacks of coins  
 problem

106. Is the multiplicity of 1  
 as an eigenvalue of  $A$  a  
 flow conjugacy inv. of  $\Sigma_A$ ?  
 How about  $\prod_{i=1}^n (1-\lambda_i)$  no

107. Embed auto. cpt gps as  
 basic sets

108. Let  $gp. G$  be given by  
 generators  $S$  with relations

Consider

$$V = \{x : x \in \prod_0^{\infty} (S \cup S^{-1}), x$$

reduced or  $\prod_{i=0}^{\infty} x_i$

What is  $V$ ? intrinsic ergodic  
 Entropy = ?

109. (Thom) Look at Markov part. on  
 $T^n$  when all  $\lambda_i$  distinct + real

110. Top. entropy of Frobenius  
 map of algebraic variety  $V$   
 - related to  $\dim V - \deg V$   
 (Zeta fns. rad. of conv.)  
 Relation to zeta fns.  
 + Weil conjectures



$\wedge$ , 122. ~~homocycle flow L.B?~~  
 (Katok-O system)  $S, T, L.B$   
 $+ 0$ -entropy  $\Rightarrow S$  induced.  
~~In Bulltop - then does~~

122. Fibration thin. for  $h(x)$ ?  
 $M = \bigcup_x N_x$ ,  $N_x$  manifolds  
 $f(N_x) = N_x$ . Then  
 $s(f_x \text{ on } M) \leq \sup_x (s_x \text{ on } N_x)$ ?

If  $f|N_x$  isometry, does  
 $s(f_x \text{ on } M) = 1$ ? Are all  
 distal diffeos built up this  
 way - i.e. extensions where  
 homology works.

123. cancellation thm. for  
 $2$  basic sets - analogue  
 of cobordism theory?

124. well-conv. for basic sets

125. Embedding alg. variety  
 over  $F_p$  in a basic set.

126. Put orientation into  $S$ -fun.  
 of flow? what should  
 $s(x)$  be?  $S(x)$  depends  
 only on  $H_x$  of  $(M, M_{-1})$ .

~~127. homocycle flow L.B?~~

128.  $R_2, R_3$  suspensions under  
 bounded fcn. isomorphisms?  
 good flow case

129.  $h(f)$  as rate of growth of  
 eigenfn. for operator in  
 momentum space - Fourier  
 transform of Laplacian on manifold?

130. Which surfaces +  
which homotopy classes  
of homeo admit

- ① expansive homeo
- ② distal homeo

131.  $h(f)$  given by per.  
pts. for generic  
 $C^1$  map  $I \rightarrow I$   
(acts map)

132.  $\dot{X} = Q(X)$  on  $\mathbb{R}^3$   
A quadratic. Is there  
a ~~crit~~ condition on  
coefficients which  
guarantees a homoclinic  
pt. (complicated)

attractor). Eg. like bi  
Rayolds no

133.  $C^\infty$  diffeo. of 2-disk  
preserving smoothness  
with  $h_u > 0$  (ergod  
one): calculate  
Entropy of Hamiltonian  
systems in Arnold-Avez

134.  $\phi: S^1 \rightarrow S^1$  cpl. pos.

① entropy  $> 0$   
 $\Rightarrow \exists$  cpl.  $\mu$ -in

② ent = 0 + ergodic  $\Rightarrow$   
 $\# N$ .


135. Max. + geod. for  $\Gamma = SL(3, 2)$ .  
 - min. vol. closed  
 - ergodic dyn. (old fac?)

136. For most  $C^2$  maps  $f: [0, 1] \rightarrow [0, 1]$   
 $\forall \epsilon > 0$  hyp. sets  $A \subset [0, 1]$   
 s.t.  $h(f|_A) > h(f) - \epsilon$ .

137. Ergodic smooth maps of



Prod.  $A^2 B$  of Dehn twists?  
 $h(f) > 0 = h(f)$ ?

138. Billiards on   
~~What~~ example where ergodic

poly. growth  
 139. Hyperfinite fol. is  
 hyperfinite? Is adding  
 $W^{ws}$  on  $\Sigma_{g_0, 13}^+$  Borel  
 hyperfinite?

140.  $g: G/\Gamma \rightarrow G/\Gamma$  alg. - from  
 auto of  $G$ . Does  $\forall g \Rightarrow h(f) \geq h(g)$ ?  
 Entropy conj. on indmanifolds?

141.  $g: G/\Gamma \rightarrow G/\Gamma$  auto.  
 Is  $G/\Gamma(g \text{ on } \Gamma)$  an alg. no.?

142. Adding machines not an  
 inv. set for  $C^2$  maps of disks?  
 w-disk?

143. G.R(x) analg. integer  
for  $\alpha$  auto. of solvable  
gp.

(144.)  $f: [0,1] \rightarrow \mathbb{C}^2$ , cont.  
Yes. w. nondog. Then  $\forall \epsilon > 0$   
L.S.  $f$  inv. expanding set  
 $X_\epsilon$  s.t.  $h(f|X_\epsilon) > h(f) - \epsilon$ .

(145.) Does  $h(f) = \log \lambda$  for  
Thurston's quasi-Anosov  $f$ .

Yes

(146.) (a) s.s. finite type have good  
quotient w.r.t. fixed pt.

(b) Given per. pt.  $p \in \Sigma$ , is  
there a Markov part. with

(c)  $p$  in interior  
If  $\delta \in \neq \emptyset$  does  $\delta \in$  contain  
a per. orbit

(d) 2 s.s. f.t. with same  
entropy  $\Rightarrow$  common  
good quotient

(e)  $\Sigma_A, \Sigma_B$  aperiodic,  $h(\Sigma_A) < h(\Sigma_B)$   
does  $\Sigma_A | \Sigma_A$  embed into  $\Sigma_B$ .

147. inv. distribution for good  
Anosov flow — approximated?  
(-Kardar) by period orbit measures?

148. Kleiman g.p. limit set  
 $\lambda$  — specification when  
all are parabolic or elliptic  
elts.

149. (Handel)  $\exists$  cross-section  
unif. for all min. sets of flow  
 $\rightarrow \exists$  cross-section



150. Is there an expansive homeomorphism of  $S^2$ ?

151.  $\underline{x} \sim y$  on  $\Sigma_A^+$  if  $\tau^n x = \tau^m y$  some  $n, m > 0$ .  
Find inv. of  $(\Sigma_A^+, \sim)$ , top.

152.  $f: M \rightarrow M$   $\infty$  Anosov  
 $f \in C^r$  and

$$f(x) = u(x) - u(Tx).$$

Does  $u \in C^r$ ? ( $r \geq 2$ )

153.  $\phi_t: T_1 M \rightarrow T_1 M$  Anosov  
geod. flows and

$V: M \rightarrow \mathbb{R}$  s.t.  $SV(\pi \phi_t x) = 0$   
on every closed geod.  
Does  $V \equiv 0$ ?

154. Geod. flows on surfaces  
 $h_u > 0$  if  $g \geq 2$

155. (Thurston-Sullivan?) Are all smooth actions of  $\mathbb{Z}$  on  $S^1$  (which are top. conj. to a standard one) diff. conj. to a standard one?

156. Kleinian gp.  $\Gamma$ .  
h.d.  $\lambda(\Gamma) \leq 2$  if  $\lambda(\Gamma) \neq S^2$

157. on closure  $\bar{T}$  of Teichmüller space.  $\phi_t$  param.

$$\bar{T} \times \Sigma_A^+ \rightarrow S^2$$

s.t. image  $(t, \Sigma_A^+) = \lambda(\Gamma_t)$

h.d.  $(\Gamma_t)$   $\phi_t$  in  $t \in \bar{T}$ ?